CHAPTER-3

Design of Shaft & Axle -

शाफ्ट एवं एक्सल सामान्यतः गोलाई आकार के cross section में पाये जाते हैं। यह (शाफ्ट एवं एक्सल) मशीन के घूमते वाले पार्टों को support करते हैं जैसे कि pulley, gears, flywheels etc. एक्सल सिर्फ load को सपोर्ट करते हुये या तो घूमता है अथवा fixed रहता है। जिसके कारण एक्सल में पड़ने वाला load bending momement मुख्यतः होता है व सिर्फ Bending stresses ही इस पर लगते हैं। जबकि शाफ्ट ना सिर्फ rotating moment को भी support करता है। जिसकी वजह से शाफ्ट पर bending व torsional moment लगाता है। जिससे इस पर bending व torsional stresses लगाते हैं।

Geometrical construction के classification अनुसार शाफ्ट दो तरह के होते हैं - (1) Straight Shaft.
(2) Crank Shaft.

Shaft का दूसरा क्लासिफिकेशन
(1) Prime Mover Shaft - Turbine shaft, Motor shaft, Engine Shaft.
(2) Machine Shaft.
(3) Power Transmission shaft - line shaft, Counter shaft, jack shaft etc.

शाफ्ट एवं एक्सल के फेल होने के कारण -
(1) Cyclic over load मौजूद होने के कारण।
(2) Stress Concentration शाफ्ट में under cuts, machining traces, knotches or fillets, keyways, drilled holes मौजूद होने के कारण।
(3) Bearing, clearance सही नहीं देने के कारण

Q-1 A Solid shaft is subjected to a bending moment of 3.46 kNm and torsional moment of 11.5 kNm. The shaft is made of C-45 steel and factor of safety is 6. Determine the diameter of shaft.

Solution:- Check in data book for C-45 page no. - 6.
Table 1 given Tensile Strength MN/m^2
it gives C-45 is between $618 - 696\ MN/m^2$
take any value between
Let say $690\ MN/m^2$

So $\sigma_b = \dfrac{\text{Ultimate tensile stress}}{\text{Factor of Safety}}$

So $= \dfrac{690}{6}$

$\boxed{\sigma_b = 115 \text{ MN/m}^2 \text{ or MPa}}$

Ultimate Shear stress usually $= 0.75 \times$ Ultimate Tensile stress

$= 0.75 \times 690$

$\boxed{\text{Ultimate Shear Stress} = 517.5 \text{ MPa or MN/m}^2}$

$\tau = \dfrac{\text{Ultimate Shear Stress}}{\text{Factor of Safety}}$

$= \dfrac{517.5}{6}$

$\boxed{\tau = 86.0 \text{ MPa or MN/m}^2}$

We shall design the shaft both for equivalent twisting moment and equivalent bending moment.

$\Rightarrow$ Equivalent Twisting Moment

$\boxed{M_{te} = \sqrt{M_b^2 + M_t^2}}$

$= \sqrt{(3.46 \times 10^3)^2 + (11.5 \times 10^3)^2}$

$\boxed{M_{te} = 12 \text{ kNm}}$ or $\boxed{T = 12 \text{ kNm}}$

Maximum shear stress under twisting moment T

$\boxed{\tau = \dfrac{16T}{\pi d^3}}$ – Eq (9.1b)

$\boxed{T = \dfrac{\pi}{16} d^3 \tau}$

$\Rightarrow \dfrac{\pi}{16} \times d^3 \times 86 = 12 \times 1000$

$\boxed{d^3 = \dfrac{12 \times 1000 \times 16}{\pi \times 86}}$

$\boxed{d = 0.0893 \text{ m} = 89.3 \text{ mm}}$

Check page No.- 76 for nearest standard diameter of shaft in mm $\boxed{d = 90 \text{ mm}}$

$\Rightarrow$ Equivalent Bending moment =

$\boxed{\cancel{M_{bf} = \sqrt{M_b^2 + M_t^2}}}$

$\boxed{M_{be} = \dfrac{1}{2}\left(M_b + \sqrt{M_b^2 + M_t^2}\right)}$

$= \dfrac{1}{2}(3.46 + 12) = 7.73 \text{ kNm}$

$\boxed{M_{be} = 7.73 \text{ kNm}}$ or $\boxed{M = 7.73 \text{ kNm}}$

Maximum Bending stress under Bending moment M

$$\boxed{\sigma_b = \frac{32M}{\pi d^3}}$$ 9.2(b) from Data Book

$$d^3 = \frac{32 \times 7.73}{\pi \times 115}$$

$$\boxed{d = 0.088 m}$$

$$\boxed{d = 88 mm}$$

From check 76 No. standard diameter of shaft (mm)

$$\boxed{d = 90 mm}$$

Diameter of the shaft is to be taken as 90mm.

Q-2 A transmission shaft with keyways is subjected to a maximum torsional moment of 750 Nm and a maximum bending moment of 1200 Nm. The loads are suddenly applied, minor shocks are encountered, and the allowable shear stress is 42 MPa. Find the shaft diameter.

Answer: From Data Book page No.- 77 equation no. 9.5

Given
$M = 1200 Nm$
$T = 750 Nm$
$\tau = 42 MPa$

$$\boxed{d^3 = \frac{16 F.S}{\pi \tau_{yp}} \sqrt{(C_m M)^2 + (C_t T)^2}}$$

or we can write the equation

$$\boxed{d^3 = \frac{16}{\pi \tau} \sqrt{(C_m M)^2 + (C_t T)^2}}$$

From table 9.1 in page No. 77
Check in rotating shaft suddenly applied load, minor shock value for $C_m = 1.5 - 2.0$, $C_t = 1.0 - 1.5$
take $\boxed{C_m = 2.0}$ $\boxed{C_t = 1.5}$

$$d^3 = \frac{16}{\pi \times 42} \sqrt{(2 \times 1200)^2 + (1.5 \times 750)^2}$$

$\boxed{d = 0.0683 m}$ or $\boxed{d = 68.3 mm}$

Check for 76 page no. of design data book check for diameter (nearest for the same above) $\boxed{d = 71 mm}$.
{We will not select the dia (70) as it is second choice}

Q-3 Determine the diameter of a hollow shaft with a ratio of $\frac{d_i}{d_o}$ of 0.8, capable of transmitting 300 kW at 225 rev/min when subjected to a maximum bending moment of 5500 Nm. The load is suddenly applied with minor shocks for torsional moment the bending moment is steady, and the allowable shearing stress is 56 MPa.

Solution - Given $K = \frac{d_i}{d_o} = 0.8$, rpm = 225 rpm

$P = 300\,kW$, $M = 5500\,Nm$

$\tau = 56\,MPa$ or $56\,N/m^2$

To find out T

$$\boxed{P_{(kW)} = \frac{2\pi N T}{60}}$$

$$\boxed{T = \frac{60 \times 10^3}{2\pi} \times \frac{P(kW)}{N}}$$

$$\boxed{T = 9550 \times \frac{P(kW)}{N}}$$

$$T = \frac{9550 \times 330}{225}$$

$$\boxed{T = 12.74\,kNm}$$

For Hollow shaft for given bending moment and torsional moment use on page No.- 76 equation No.- 9.4

$$d_o^3 = \frac{16\,F.S.}{\pi\,\tau_{yp}(1-K^4)}\sqrt{\left[C_m M + \frac{\alpha p d_o (1+K^2)}{8}\right]^2 + [C_t T]^2}$$

or

$$\boxed{d_o^3 = \frac{16}{\pi(1-K^4)\tau}\sqrt{[C_m M]^2 + [C_t T]^2}}$$ — New equation

Eliminate values - $\frac{\alpha p d_o (1+K^2)}{8}$

and

$$\boxed{\tau = \frac{\tau_{yp}}{F.S.}}$$

Putting value in New equation -

$$d_o^3 = \frac{16}{\pi \times 56 \times (1-(0.8)^4)}\sqrt{(1.5 \times 5.5 \times 10^3)^2 + (1.5 \times 12.74 \times 10^3)^2}$$

C_m and C_t is taken as 1.5 from table 9.1 from page No.- 77

$$d_o^3 = \frac{16}{\pi \times 56 \times (1-(0.8)^4)}\sqrt{(1.5 \times 5.5 \times 10^3)^2 + (1.5 \times 12.74 \times 10^3)^2}$$

$\boxed{d_o = 147.7\,mm}$ The nearest d_o = 150 mm from page 76 from standard dia list.

$$\frac{d_i}{d_o} = 0.8 \text{ given}$$

$$d_i = 0.8 \times 150$$

$$\boxed{d_i = 120mm}$$

Question-4 A popeller shaft is requiered to transmit 45 kW power at 500 rpm. It is a hollow shaft, having an inside diameter 0.6 times of outside diameter. It is made of plain carbon steel and the permissible shear stess is 84 N/mm². Calculate the inside and outside diameters of the shaft.

Given $P = 45\ kW$, $N = 500\ rpm$, $\tau = 84\ N/mm^2$

$d_i = 0.6 d_o$ or $\boxed{d_i/d_o = 0.6}$ or $\boxed{K = 0.6}$

Torque transmitted by shaft

$$T = \frac{60 \times 10^6 (kW)}{2\pi N} = \frac{60 \times 10^6 (45)}{2\pi (500)}$$

$$= 859436.69\ N\text{-}mm$$

Inner and Outer diameters of shaft

$$\boxed{\frac{d_i}{d_o} = 0.6}$$

From equation 9.1b from design data book

$$\boxed{\tau = \frac{16 T d_o}{\pi (d_o^4 - d_i^4)}} \quad - 9.1(a)$$

$$= \frac{16 T d_o}{\pi d_o^4 \left(\frac{d_o^4}{d_o^4} - \frac{d_i^4}{d_o^4}\right)}$$

$$= \frac{16 T d_o}{\pi d_o^4 (1 - K^4)}$$

$$\boxed{\tau = \frac{16 T}{\pi d_o^3 (1 - K^4)}}$$

$$d_o^3 = \frac{16 T}{\pi \times \tau (1 - K^4)}$$

$$= \frac{16 \times 859436.69}{\pi \times 84 \times (1 - (0.6)^4)}$$

$$\boxed{d_o = 39.12 m}$$

Given $\boxed{\dfrac{d_i}{d_o} = 0.6}$

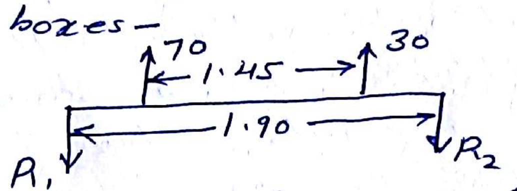
$$d_i = 0.6\,(39.12)$$

$$\boxed{d_i = 23.47\,mm}$$

Question-5 In a railway wagon, the maximum load on a pair of wheels is 100 kN: one wheel take 70 kN the other 30 kN. The distance between the rails is 1.45 m and between the centres of the axle boxes is 1.90 m. Find the diameter of the axle at the wheel. Safe stress is 77 MPa.

Solution- Let R_1 and R_2 denote the loads on the axle boxes -

Given $\sigma_b = 77\,MPa$
$= 77\times10^6\,N/m^2$



This is not included in the calculation

$$R_1 \times 1.90 = 70\times1.675\{(1.45+0.225)\} + 30\times0.225$$

$$R_1 = \frac{124}{1.90} = 65.2\,kN$$

$$R_1 + R_2 = 100\,kN \;(\text{Given})$$

$$R_2 = 100 - R_1 = 100 - 65.2 = 34.8\,kN$$

Maximum bending moment on the wheel

$$M_b = 65.2\times0.225 = 14.68\,kNm$$

or

$$\boxed{M = 14.68\,kNm}$$

Using equation No.- 9.2 (b)

$$\boxed{\sigma_b = \frac{32M}{\pi d^3}}$$

$\cancel{14.68\times10^3 \times 32}$

$$d^3 = \frac{32\times M}{\pi\times\sigma_b}$$

$$= \frac{32\times14.68\times1000}{\pi\times77\times10^6}$$

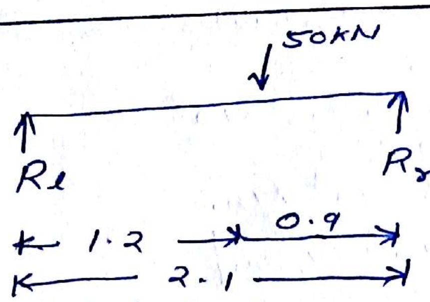
$$= 1.94\times10^{-3}\,m^3$$

$$\therefore \boxed{d = 0.1248\,m \cong 125\,mm}$$

Checking with the standard diameter of shaft 125 mm is the right answer as table provides the 125 mm data.

Question-6 An axle is supported on two end journals and carries a load of 50 kN at a point 1.2 metre from one journal and 0.9 metre from the other. Determine maximum diameter of the axle, if the stress allowed is 63.0 MPa.

Solution



Let the load be located 1.2 metre from the left journal

∴ Reaction on the left journal.

$$R_l = \frac{50 \times 0.9}{2.1} = 21.4 \text{ kN}$$

$$R_r = 50 - 21.4 = 28.6 \text{ kN}$$

Maximum bending moment will occur at the load point

$$\sigma_b = M_b = R_l \times 1.20 = R_r \times 0.9$$

$$= 21.4 \times 1.2 = 25.68 \text{ kNm}$$

$$\boxed{\sigma_b = 25.68 \text{ kNm}}$$

$$\boxed{\sigma_b = \frac{32M}{\pi d^3}}$$ For Solid shaft equation 9.2(b) page No.- 76

$$d^3 = \cancel{\frac{\sigma_b \pi}{32 M}} \quad \frac{32M}{\pi \sigma_b}$$

$$= \frac{32 \times 25.68 \times 10^3}{\pi \times 63 \times 10^6}$$

$$= 4.152 \times 10^{-3} \text{ m}^3$$

$$\boxed{d = 0.1607 \text{ m}}$$

The nearest equivalent diameter at page No.- 76 is

$$\boxed{d = 160 \text{ mm}}$$

Question-7 ~~Ga~~ एक ठोस शाफ्ट 800 च.पु.मि. पर 0.5 MW शक्ति संचारित करता है। अधिकतम आघूर्ण, औसत आघूर्ण से 20% अधिक है। सुरक्षित कर्तन प्रतिबल 60N/mm² लेकर शाफ्ट का व्यास ज्ञात कीजिए।

Given $P = 0.5 \text{ MW} = 0.5 \times 10^6 \text{ W}$

$N = 800 \text{ rpm}$

$T_{max} = 1.2 \, T_{mean}$

$\tau = 60 \text{ N/mm}^2$

∴ $$P(\text{kW}) = \frac{2\pi N T}{60}$$

$$T = \frac{60\times10^3\,(P)}{2\pi\times800}$$

$$= \frac{9550\times(P\,in\,kW)}{800}$$

$$= \frac{9550\times\cancel{10}0.5\times10^3}{800}$$

$$= 5.968\times10^3\ Nm$$

$$\boxed{T_{mean} = 5.96\times10^6\ Nmm}$$

$T_{max} = 1.20\,T_{mean}$ (Given)

$$= 1.20\times5.96\times10^6$$

$$\boxed{T_{max} = 7.164\times10^6\ Nmm}$$

Maximum shear stress under twisting moment T from equation 9.1 (b)

$$\tau = \frac{16T}{\pi d^3}$$

$$\boxed{d^3 = \frac{16T}{\pi\tau}}$$

$$d^3 = \frac{16\times7.164\times10^6}{\pi\times60}$$

$$d^3 = 0.608\times10^6$$

$$\boxed{d = 84.71\ mm}$$

The nearest standard dia from page 76 is 85 mm. So answer is $\boxed{d = 85\ mm}$

Question-8 The transmission shaft having at one end helical and at one end bevel gears. The CTC (Centre to Centre) distance is 800 mm. For this shaft the permissible angle of twist is 0.25° per metre length. The modulus of rigidity for the shaft material is 79300 N/mm². Calculate

(i) The permissible angle of twist between helical and bevel gears and

(ii) The shaft diameter on the basis of torsional rigidity.

Compare the results of these two examples and comment on the results.

Answer - Given $G = 79300\ N/mm^2$

$\theta = 0.25°$ per metre

Permissible angle of twist for shaft

$$\theta = 0.25\left(\frac{800}{1000}\right)$$

$$\boxed{\theta = 0.2°}$$

From equation at 3.3 at page No.-17

* $$\boxed{\frac{T}{I_p} = \frac{G\theta}{l}}$$

Where T = Torque applied at the section of shaft.

I_p = Polar moment of Inertia of the shaft

From page No.-16 polar moment of inertia of circular section

$$\boxed{I_p = \frac{\pi}{32} d^4}$$

$$\therefore \frac{T \times 32}{\pi \times d^4} = \frac{G\theta}{l}$$

$$d^4 = \frac{32}{\pi} \frac{T \times l}{G \times \theta}$$

G = modulus of rigidity of shaft material

θ = Angle of Twist of the shaft in radians.

l = Length of the shaft.

$$\therefore \theta = \frac{T \times l}{I_p \times G}$$

Converting θ from radians to degree

$$\theta = \frac{180}{\pi} \times \frac{T \times l}{I_p \times G}$$

$$\theta = \frac{180}{\pi} \times \frac{32 \times T \times l}{\pi d^4 \times G}$$

$$\theta = \frac{180 \times 32}{\pi \times \pi} \frac{T \times l}{d^4 \times G} \quad \left\{ \frac{180 \times 32}{\pi \times \pi} = 583.61 \simeq 584 \right\}$$

* $$\boxed{\theta = 584 \frac{T \times l}{d^4 \times G}}$$ → * You must remember the formula

$$\boxed{d^4 = 584 \frac{T \times l}{\theta \times G}}$$

Substituting the data given

$$= \frac{584 \ (144000)(800)}{(79300)(0.2)}$$

$$\boxed{d = 45.38 \text{ mm}}$$

Therefore, it is necessary to design the shaft on the basis of strength as well as rigidity

Question-9 A factory line shaft is 4.5 metre long and is to transmit 75 kW at 200 rpm. The allowable fibre stress in shear is 49 MPa, and the maximum allowable twist is 1 deg. in a length of 20 diameters. Determine the required shaft diameter.

Solution The twisting moment on the shaft is

$$P = \frac{2\pi N T}{60}$$

$$T = \frac{P(kW) \times 60 \times 10^3}{N \times 2\pi}$$

$$= \frac{9550 \times 75}{200}$$

$$\boxed{T = 3580 \, Nm}$$

Given
$\tau = 49 mp$
$= 49 N/m^2 \times 10^6$
rpm = N = 200 rpm
P = 75 kW

$\Rightarrow$ Diameter of shaft on the Strength basis

$$\boxed{T = \tau \times \frac{\pi}{16} d^3}$$

Page - 76
equation - 9.1(b)

$$d^3 = \frac{T \times 16}{\pi \times \tau}$$

$$= \frac{3580 \times 16}{\pi \times 49 \times 10^6}$$

$$\boxed{d = 0.072 m}$$

$\Rightarrow$ Diameter of shaft on the basis of rigidity

$$\boxed{\theta = \frac{T l}{I_p G}}$$

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Equation (3.3)

$\theta = 1^\circ = \frac{\pi}{180}$ rad

$l = 20d$

$I_p = \frac{\pi}{32} d^4$

$G = 84 \times 10^9 = 84 \, GPa$

$$\frac{\pi}{180} = \frac{3580 \times 20d \times 32}{\pi d^4 \times 84 \times 10^9}$$

$$\therefore d^3 = \frac{3580 \times 20 \times 32 \times 180}{\pi \times \pi \times 84 \times 10^9}$$

$$\boxed{d = 0.079 m}$$

In this case the shaft diameter required for strength is less than required for torsional rigidity and so the shaft diameter should be made 0.079m that is 80mm to the nearest size.